

Exercice 2

1) Déterminer la loi de probabilité du couple (X, Y)

$$X = \{1, 2, 3\} \quad Y = \{1, 2, 3\}$$

$$\bullet P(X=1, Y=1) = \frac{1}{6} \times 0 = 0$$

$$\bullet P(X=1, Y=2) = \frac{1}{6} \times \frac{2}{5} = \frac{2}{30}$$

$$\bullet P(X=1, Y=3) = \frac{1}{6} \times \frac{3}{5} = \frac{3}{30}$$

$$\bullet P(X=2, Y=1) = \frac{2}{6} \times \frac{1}{5} = \frac{2}{30}$$

$$\bullet P(X=2, Y=2) = \frac{2}{6} \times \frac{2}{5} = \frac{4}{30}$$

$$\bullet P(X=2, Y=3) = \frac{2}{6} \times \frac{3}{5} = \frac{6}{30}$$

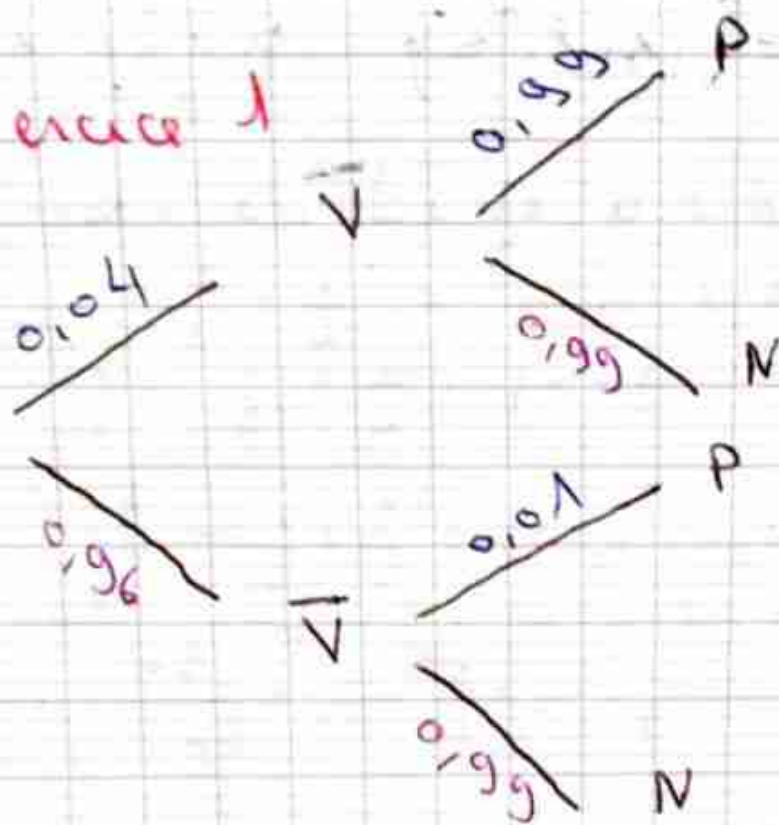
$$\bullet P(X=3, Y=1) = \frac{3}{6} \times \frac{1}{5} = \frac{3}{30}$$

$$\bullet P(X=3, Y=2) = \frac{3}{6} \times \frac{2}{5} = \frac{6}{30}$$

$$\bullet P(X=3, Y=3) = \frac{3}{6} \times \frac{3}{5} = \frac{9}{30}$$

$\frac{X}{Y}$	1	2	3	
1	0	$\frac{1}{15}$	$\frac{1}{10}$	$\frac{1}{6}$
2	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{2}{5}$	$\frac{1}{3}$
3	$\frac{1}{10}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{1}{2}$
	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	

Exercice 1



$$P(V) = 0,04 \text{ et } P(P|V) = 0,99, P(N|\bar{V}) = 0,01$$

on veut calculer

$$P(\bar{V}|P) = \frac{P(\bar{V} \cap P)}{P(P)} = \frac{P(\bar{V}) P(P|\bar{V})}{P(P)}$$

$$\text{et } P(P) = P(P \cap V) + P(P \cap \bar{V}) \\ = P(V) P(P|V) + P(\bar{V}) \times P(P|\bar{V})$$

$$\text{d'où } P(\bar{V}|P) = \frac{P(\bar{V}) P(P|\bar{V})}{P(\bar{V}) \times P(P|\bar{V}) + P(V) \times P(P|V)}$$

$$\text{AN: } P(\bar{V}|P) = \frac{0,96 \times 0,101}{0,96 \times 0,101 + 0,04 \times 0,99}$$

$$P(\bar{V}|P) = 0,19$$

Exercice 3

1) Probabilité que le service réuni soit du joueur B

$$P_R(B) = P(B/R) = \frac{P(R|B)}{P(R)} = \frac{P(R|B) \times P(B)}{P(R)}$$

$$P_R(B) = \frac{P(R|B) \times P(B)}{P(B) \times P(R|B) + P(B) \times P(R|A)}$$

$$= \frac{0,6 \times \frac{2}{3}}{\frac{2}{3} \times 0,6 + \frac{1}{3} \times 0,9}$$

$$= \frac{\frac{2}{3} \times 0,6}{\frac{2}{3} \times 0,6 + \frac{1}{3} \times 0,9}$$

$$2) P_R(B) = \frac{P \times P(B)}{P(B) \times P + P(A) \times P(R|A)} = \frac{\frac{1}{2} \times \frac{2}{3}}{\frac{1}{2} \times \frac{2}{3} + \frac{1}{3} \times 0,9}$$

$$P_R(B) = 0,52$$

$$3) P(R|B) = P(R|A) = P'$$

$$P_R(B) = \frac{P' \times P(B)}{P(B) \times P' + P(A) \times P'} = \frac{P(B)}{P(B) + P(A)}$$

$$\text{AN: } P_R(B) = 0,66$$

2) En deduire la loi de probabilité de $S = X + Y$

$$S = X + Y = \{2, 3, 4, 5, 6\}$$

S	2	3	4	5	6
P_{X+Y}	0	$\frac{2}{15}$	$\frac{4}{15}$	$\frac{2}{5}$	$\frac{1}{5}$

$$\begin{aligned} \bullet P(S=2) &= 0 & \bullet P(S=3) &= \frac{1}{15} + \frac{1}{15} = \frac{2}{15} \\ \bullet P(S=4) &= \frac{1}{15} + \frac{1}{10} + \frac{1}{10} = \frac{4}{15} \\ \bullet P(S=5) &= \frac{1}{5} + \frac{1}{5} = \frac{2}{5} \\ \bullet P(S=6) &= \frac{1}{5} \end{aligned}$$

3)

$$\begin{aligned} E(S) &= \sum_{i=1}^n p_i \cdot x_i \\ &= 0 \cdot 2 + \frac{2}{15} \cdot 3 + \frac{4}{15} \cdot 4 + \frac{2}{5} \cdot 5 + \frac{1}{5} \cdot 6 \\ &= \frac{14}{3} \end{aligned}$$

$$V(S) = \sum_{i=1}^n (x_i - \bar{x})^2 p_i$$

$$\begin{aligned} &= \left(2 - \frac{14}{3}\right)^2 \times 0 + \left(3 - \frac{14}{3}\right)^2 \times \frac{2}{15} + \left(4 - \frac{14}{3}\right)^2 \times \frac{4}{15} \\ &\quad + \left(5 - \frac{14}{3}\right)^2 \times \frac{2}{5} + \left(6 - \frac{14}{3}\right)^2 \times \frac{1}{5} \end{aligned}$$

$$V(S) = \frac{8}{3} \approx 0,88$$